A comparative study on the variation of soil temperature at different depths

Rezuana Islam

Abstract—For the sake of energy efficiency in building heat pump emerges as a fruitful solution. Geothermal heat pump requires knowing soil temperature at different depths. In a regression analysis conducted for Dhaka division of Bangladesh Islam et al. [1] estimated regression equations for determining soil temperature at different depths from air temperature. The equations are supposed to be applicable to other similar climatic regions of the world. Level of reliability of these equations is claimed to be strong up to 10cm depth. In this paper we have first of all derived equations for determining variations of soil temperature at different depths using the said regression equations. Analogue of these equations is then derived using the sinusoidal temperature model which was obtained by solving the partial differential equation of heat flow in soil. The two sets of equations are then compared. Regression equations estimated in Islam et al.[1] relating air temperature and soil temperature, are not applicable below 10cm depth whereas the sinusoidal temperature model works at all depths. It seems that this study can help estimating soil temperature from air temperature at all depths. It will further help architects and other bodies concerning building construction designing energy efficient building.

Index Terms— Energy efficient building design, Regression analysis, Sinusoidal temperature function, Air temperature, Soil temperature, Variation of soil temperature, Damping depth, Amplitude.

---

1 Introduction

NOWADAYS global warming is a burning issue and energy efficient building design is a big challenge for architects. From economical, environmental and social point of view current trends in energy supply and use are unsustainable. For decades, space heating and cooling (space conditioning) required for more than half of all residential energy consumption and without crucial action, global energy-related greenhouse gas (GHG) emissions will more than double by 2050. Heat pump is a successful solution which requires less energy for heating or cooling a building. For climates with moderate heating and cooling needs, heat pumps offer an energy-efficient alternative to furnaces and air conditioners. To design a geothermal heat pump soil temperature at different depths is needed to be found out. Geothermal (ground-source or water-source) heat pumps by transferring heat between building and the ground or a nearby water source achieve higher efficiency. Geothermal heat pump has low operating costs as it takes advantage of relatively constant ground or water temperatures. It can reduce energy use by 30%-60%.

Soil is a loose combination of organic and inorganic materials that covers the land surfaces of the earth. Soil temperature has its importance in many scientific instances such as architectural designing etc. Soil temperature depends on climatic conditions and doesn't follow the same pattern throughout the year. This is because of the solar thermal energy and the energy balance between solar thermal energy and ground thermal energy. A number of attempts have been made to estimate soil temperature from air temperature and to estimate the variation of soil temperature at different depths and times of the year. All these estimations are approximate and not exact. So there are uncertainties in their level of reliability. In this study we have derived two sets of equations describing the dependence of soil temperatures at different depths using two different approaches. A comparative study is then provided.

• *Rezuana Islam, Lecturer, Department of Architecture, Chittagong University of Engineering & Technology, Chittagong, Bangladesh.*
E-mail: 124arch0709@gmail.com

The rest of this paper is organized in the following way. In section 2 a brief review of a regression analysis intended to estimate the dependence of soil temperature on air temperature is provided. Section 3 provides a review of the modeling of soil temperature with depth based on the heat flow equation in soil. Section 4 is original and provides the main result of this paper. Here we have derived two sets of equations for determining soil temperature at different depths using regression equations presented in section 2 and the sinusoidal temperature model presented is section 3 and a comparative study of the two sets of equations is provided. Finally in section 5 some concluding remarks are given.

2 Correlation Between Air Temperature and Soil Temperature

In a study on the correlation between air temperature (x^o C) and soil temperature (y^o C) Islam et al. [1] speculated regression equations relating x and y. The study was conducted for Dhaka Division of Bangladesh and is supposed to be applicable to other similar climatic regions of the world. The general form of the equation is linear and the analysis indicated strong correlation between the variables x and y up to 20cm depth. Coefficient of correlation (r) at 5 cm and 10cm depths was found to be 0.93. At 20 and 30cm depths coefficient of correlation (r) were found to be 0.870 and 0.63 respectively. This value of r decreases rapidly with the increase of depth.

To validate the correlation a series of data was collected for the two variables x and y for 10 years from 2003 to 2012. To demonstrate the linear relation between x and y scatter plots for the whole 10 years data were generated for the depths 5cm, 10cm, 20cm and 30cm. The equations from regression analysis for the said depths are found to be as follows,

Equation for 5cm depth: $y = 3.83 + 0.9x$ (1)

Equation for 10cm depth: $y = 6.224 + 0.842x$ (2)

International Journal of Scientific & Engineering Research, Volume 8, Issue 7,July 2017
ISSN 2229-5518

Equation for 20 cm depth: $y = 9.871 + 0.708x$ (3)

Since the coefficient of regression decreases with the increase of depth we find that the two variables *x* and *y* gradually looses linearity with the increase of depth. Linearity relation is almost lost at 30cm depth.

3 Soil Temperature Model

Annual variation of daily average soil temperature at various depths can be estimated by using a sinusoidal function which was derived by solving the following heat flow equation (Hillel [2]; Marshall and Holmes [3]; Wu and Nofziger [4])

$$\frac{\partial T(z,t)}{\partial t} = D_h \frac{\partial^2 T(z,t)}{\partial z^2} \quad (4)$$

where, *z* is soil depth, *t* is time, T(*z*,*t*) is the soil temperature at time *t* and depth *z* and $D_h = k/c$ is the thermal diffusivity (k is the thermal conductivity and c is the volumetric heat capacity). Using the boundary conditions $(0,t) = T_a + A_0 \sin \omega t$, $T(\infty, t) = T_a$, solution of equation (4) can be obtained as follows,

$$T(z,t) = T_a + A_0 e^{-z/d} \sin(\omega t + \tfrac{z}{d}) \quad (5)$$

where T_a is the average soil temperature (ºC), A_0 is the annual amplitude of the soil surface temperature (ºC), d is the damping depth(meter) of daily fluctuation and *T*(*z*,*t*) is the soil temperature at time *t* (day number of the year) and soil depth *z*(meter). Damping depth *d* is given by $d = \sqrt{\frac{2D_h}{\omega}}$, where $\omega = \frac{2\pi}{365} day^{-1}$. Solution (5) of the equation (4) is not affected with the addition of constants. Hillel took the solution in the form

$$T(z,t) = T_a + A_0 e^{-z/d} \sin[\tfrac{2\pi(t-t_0)}{365} - \tfrac{z}{d} - \tfrac{\pi}{2}] \quad (6)$$

Solution (6) can be used to determine the temperature at any given depth *z* and time *t*. In equation (6) t_0 is the lowest temperature day number of the year.

There are many models for the determination of soil temperature for different types of soil and climate having different values of D_h and t_0. B. Poudel et al. [5] constructed a model to determine the temperature for predefined depths at any day of the year for Nepal. To examine the accuracy level of the sinusoidal equation they compared the observed and calculated curves. The curves were plotted for the soil temperature against the day number of the year at the predicted depths 5cm, 10cm, 20cm and 30cm. They found good agreement among the observed and calculated values. Hence calculated values are reliable. Results showed that temperature increases with depth in winter. In summer temperature first decreases up to 10cm depth and then increases with depth.

4 Main Result

Islam et al. [1] performed a regression analysis speculating a correlation between air temperature and soil temperature. The work was concluded with the regression analysis. In the regression analysis air temperature was regarded as independent variable (*x*) and soil temperature as dependent variable (*y*). The general form of the equation was considered linear, $y = ax + b$.

Regression equations for 5cm, 10cm and 20cm depths were found to be given by equation (1), (2) and (3) respectively. Suppose for an air temperature *x*ºC temperatures at $d_1 cm$ and $d_2 cm$ depths are y_1℃ and y_2℃ respectively. Then

$$y_1 = a_1 + b_1 x \quad (7)$$
$$y_2 = a_2 + b_2 x \quad (8)$$

From (7) and (8) we obtain the following relation between y_1 and y_2,

$$y_1 = \frac{b_1}{b_2} y_2 + a_1 - \frac{a_2 b_1}{b_2} \quad (9)$$

Equation (9) is obtained by using the regression equations estimated in Islam et al. [1], which correlates air temperature and soil temperature.

Now consider the sinusoidal function (6) which describes the annual variation of daily average soil temperature at different depths,

$$T(z,t) = T_a + A_0 e^{-z/d} \sin[\tfrac{2\pi(t-t_0)}{365} - \tfrac{z}{d} - \tfrac{\pi}{2}] \quad (6)$$

where *T*(*z*,*t*) is the soil temperature at time *t* (day number of the year) and soil depth *z*(meter). Let, $Y_1 = T(z_1, t)$ and $Y_2 = T(z_2, t)$. Then from equation (6) we obtain,

$$Y_1 = T_a + A_0 e^{-z_1/d} \sin[\tfrac{2\pi(t-t_0)}{365} - \tfrac{z_1}{d} - \tfrac{\pi}{2}] \quad (10)$$

$$Y_2 = T_a + A_0 e^{-z_2/d} \sin[\tfrac{2\pi(t-t_0)}{365} - \tfrac{z_2}{d} - \tfrac{\pi}{2}] \quad (11)$$

Equations (10) and (11) can be expressed as,

$$\sin^{-1} \frac{(Y_1 - T_a)}{A_0} e^{\frac{z_1}{d}} = \frac{2\pi(t-t_0)}{365} - \frac{z_1}{d} - \frac{\pi}{2} \quad (12)$$

and, $$\sin^{-1} \frac{(Y_1 - T_a)}{A_0} e^{\frac{z_1}{d}} = \frac{2\pi(t-t_0)}{365} - \frac{z_1}{d} - \frac{\pi}{2} \quad (13)$$

respectively.

From equations (12) and (13) we obtain,

$$\sin^{-1} \frac{(Y_1 - T_a)}{A_0} e^{\frac{z_1}{d}} = \sin^{-1} \frac{(Y_2 - T_a) e^{\frac{z_2}{d}}}{A_0} - \frac{z_1 - z_2}{d} \quad (14)$$

Or, $$Y_1 = A_0 \sin[\sin^{-1} \frac{(Y_2 - T_a)}{A_0} e^{\frac{z_2}{d}} - \frac{z_1 - z_2}{d}] + T_a e^{\frac{z_1}{d}} \quad (15)$$

Equation (15) is the analogue of equation (9). Equation (15) looks very complicated. However if $\frac{(Y_1 - T_a)}{A_0} e^{\frac{z_1}{d}}$ and $\frac{(Y_2 - T_a)}{A_0} e^{\frac{z_2}{d}}$ are small equation (14) can be approximated as follows,

$$Y_1 = e^{\frac{z_2}{d}} Y_2 + \frac{(z_2 - z_1)}{d} A_0 - (e^{\frac{z_2}{d}} - e^{\frac{z_1}{d}}) T_a \quad (16)$$

Forms of equation (9) and (16) are similar.

4.1 Comparison

4.1.1 Variation of Soil Temperature at Different Depths

International Journal of Scientific & Engineering Research, Volume 8, Issue 7,July 2017
ISSN 2229-5518

Regression analysis showed strong correlation at depths 5cm and 10cm. For this reason we consider soil temperature variations at these depths. Let, $d_1 = z_1 = 5cm$ and $d_2 = z_2 = 10cm$. In approximating equation (16) the quantities $\frac{(Y_1 - T_a)}{A_0} e^{\frac{z_1}{d}}$ and $\frac{(Y_2 - T_a)}{A_0} e^{\frac{z_2}{d}}$ are supposed to be small so that Y_1 and Y_2 should be very close to T_a. We need to know the quantities $A_{0,}$ T_a and d (damping depth) for Dhaka Division. But the values of these quantities are not known. However we can choose reasonable values for these quantities. Here we choose $A_0 = 6$°C, $T_a = 24$°C and $d = 183cm$. We assume A_0 and T_a to be the same at all depths. Now we let, $Y_2 = y_2 = 25$°C. Then the equations (4) and (16) give $y_1 = 23.823$°C and $Y_1 = 25.874$°C respectively. We find that the temperature difference between y_1 and Y_1 is about 2°C. Sinusoidal temperature model predicts that as the depth increases from 5cm to 10cm soil temperature decreases from 25.874°C to 25°C whereas regression equations predict an increase of temperature from 23.823°C to 25°C.

4.1.2 Soil Temperature from Air Temperature

Section 2 assumes a linear relation between air temperature (x) and soil temperature (y),

$$y = a + bx \tag{17}$$

Sinusoidal function describing soil temperature at depth *z* and time *t* is,

$$y = T(z,t) = T_a + A_0 e^{-z/d} \sin[\omega t - \tfrac{z}{d} - \tfrac{\pi}{2}] \tag{18}$$

where, $\omega = \frac{2\pi}{365}/day$. From (17) and (18) we obtain,

$$\omega t = \sin^{-1}[\frac{(bx + a - T_a)}{A_0} e^{\frac{z}{d}}] + \frac{z}{d} + \frac{\pi}{2} \tag{19}$$

At 5cm depth equation (19) becomes,

$$\omega t = \sin^{-1}[\frac{(0.9x + 3.83 - 24}{6} e^{\frac{5}{183}}] + \frac{5}{183} + \frac{\pi}{2} \tag{20}$$

As before we assume $T_a = 24$°C, $A_0 = 6$°C and $d = 183cm$. Putting (20) in (18) we obtain,

$$T(z,x) = 24 + 6e^{-z/d} \sin[\sin^{-1}\left\{\frac{0.9x + 3.83 - 24}{6} e^{\frac{5}{183}}\right\} + \frac{5}{183} - \frac{z}{d}\ ] \tag{21}$$

where $T(z,x)$ denotes soil temperature at depth z when air temperature is x. If $x = 25$°C and $z = 10cm$ from (21) we get, $T(10,25) \approx 26.1$°C

At 10cm depth equation (18) predicts the temperature, $y = T(10,25) = 27.27$°C

Sinusoidal temperature model predicts a soil temperature at 10cm depth to be 26.1°C if the air temperature is 25°C whereas regression equation predicts this to be 27.27°C. The difference is about 1.17°C.

5 Conclusion

We have provided a comparative study on the variation of soil temperature at different depths. However in this study there are few shortcomings which should be mentioned. For such a comparison exact values of the physical constants A_0, T_a and d (damping depth) should be used. But values of these quantities are not known. This is why; we used assumed values of these constants in our calculations. Hence the results obtained may not be precise. However our intension is to construct a method of performing such a comparison. Using the assumed values of the physical constants A_0, T_a and d we have simply demonstrated the method.

One of the limitations of the regression analysis reviewed in section 2 is that, it cannot predict soil temperature from air temperature below 20cm depth. It seems from our study that this work can be extended to estimate soil temperature from air temperature using the sinusoidal temperature model so that air temperature can be used to estimate soil temperature from air temperature at any depth. We hope to provide such an extension in a future work and which will help designer and architects designing energy efficient building.

References

[1] Islam, K., Khan, A. and Islam, T., "*Correlation between Atmospheric Temperature and Soil Temperature: A Case Study for Dhaka, Bangladesh.*" Atmospheric and Climate Sciences, 5, pp.200-208, 2015.

[2] Hillel, D., "*Introduction to soil physics*", Academic Press, San Diego, CA, 1982. (Book style)

[3] Marshall, T. J. and J. W. Holmes, "*Soil Physics*", 2nd ed., Cambridge Univ. Press, New York, 1988. (Book style)

[4] Wu J., Nofziger, D.L., "*Incorporating temperature effects on pesticide degradation into a management model.*", J. Environmental Qual., 28, pp.92-100, 1999.

[5] B. Poudel, P.C. Poudel & S. Gurung, "*The modeling of soil temperature with depth*", unpublished. (Unpublished manuscript)